Quasiparticle lifetime behaviour in a simplified self–consistent T–matrix treatment of the attractive Hubbard model in 2D

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The attractive Hubbard model on a 2–D square lattice is studied at low electronic densities using the ladder approximation for the pair susceptibility. This model includes (i) the short coherence lengths known to exist experimentally in the cuprate superconductors, and (ii) two–particle bound states that correspond to electron pairs. We study the quasiparticle lifetimes in both non self–consistent and self–consistent theories, the latter including interactions between the pairs. We find that if we include the interactions between pairs the quasiparticle lifetimes vary approximately linearly with the inverse temperature, consistent with experiment.

Keywords: attractive Hubbard model; quasiparticle properties

Numerous recent experiments (*e.g.*, neutrons, ARPES, optical, NMR) have shown that in the high T_c cuprate superconductors a so–called pseudogap is present [1]. This has led to proposals that electron pairs form at temperatures well above the superconducting transition temperature. However, possibly due to phase fluctuations, a macroscopic phase coherent wave function is not formed, so superconductivity is not encountered [2].

This physics motivates our study of the attractive Hubbard model. The Hamiltonian for this system is

$$H = -t \sum_{\langle ij\rangle,\sigma} (c^\dagger_{i,\sigma} c_{j,\sigma} + \text{h.c.}) \;-\; \mid U \mid \; \sum_i n_{i,\uparrow} n_{i,\downarrow} \quad (1)$$

where the lattice sites of a 2–D square lattice are labeled by $\{i\}$, the lattice fermion operators are denoted by $c_{i,\sigma}$, and neighbouring sites are represented by $\langle ij\rangle$. For any nonzero $\mid U \mid$, two–particle bound states appear, and are physically related to a pair of electrons lowering the system's energy when they exist on the same lattice site. This is certainly the simplest example of a model Hamiltonian which allows for one to study the interactions between such electron pairs. Further, using a Gorkov derivation of the Ginzburg–Landau equations for an s–wave superconductor, one can show that for the values of $\mid U \mid /W$ that we are considering, $W = 8t$ being the (noninteracting) bandwidth, this model also reproduces the short coherence lengths of the Cooper pairs found in the high T_c cuprate superconductors [3].

We employ the Brueckner Hartree–Fock theory to solve the Bethe–Salpeter equation for the 2D attractive Hubbard model in the ladder approximation [4], reliable for this model at low electron (or hole) densities. This affords us the opportunity to investigate the influence of preformed pairs *with arbitrary lifetimes* on the normal state properties. Haussmann has argued [5] that when one solves this model in this approximation *self consistently*, one will include pair–pair interactions. These interactions will be, in first order, of a repulsive nature between different pairs.

The equations for the Green's function (G), pair susceptibility (χ), four–leg vertex function (Γ), and self energy (Σ), in a conserving approximation are well known:

$$G(\mathbf{k}, i\omega_n) = \left(G_0(\mathbf{k}, i\omega_n)^{-1} - \Sigma(\mathbf{k}, i\omega_n)\right)^{-1} \quad (2)$$

$$\chi(\mathbf{K}, i\Omega_n) = \frac{-1}{N\beta} \sum_{m,\mathbf{k}} G(\mathbf{K}-\mathbf{k}, i\Omega_n - i\omega_m) G(\mathbf{k}, i\omega_m) \quad (3)$$

$$\Gamma(\mathbf{K}, i\Omega_n) = \mid U \mid / \left(1 + \mid U \mid \; \chi(\mathbf{K}, i\Omega_n)\right) \quad (4)$$

$$\Sigma(\mathbf{k}, i\omega_n) = \frac{1}{N\beta} \sum_{m,\mathbf{q}} \Gamma(\mathbf{k}+\mathbf{q}, i\omega_n + i\omega_n) G(\mathbf{q}, i\omega_m) \quad (5)$$

where the wave vectors and Matsubara frequencies have their usual meaning. In a non self–consistent theory, one replaces the full Green's functions G in Eqs. (3,5) with the noninteracting Green's functions G_0.

For full self consistency, this set of equations has to be solved iteratively. Since such solutions are very difficult to obtain, we have investigated a simple approximation that allows for extensive numerical investigations of the resulting equations. To be specific, during the first step of the iteration process leading to self consistency, we make an approximation for the pair susceptibility as being equal to the $\mathbf{k}$–average (denoted from now on as overlined quantities, *e.g.* $\overline{\Gamma}$) of the noninteracting pair susceptibility. We only calculate the $\mathbf{k}$–averaged pair susceptibility during subsequent iterations to self consistency. This leads to the following expressions

$$\overline{\Gamma}(i\Omega_n) = \frac{1}{N} \sum_{\mathbf{K}} \Gamma(\mathbf{K}, i\Omega_n) \approx \mid U \mid / \left(1 + \mid U \mid \overline{\chi}(i\Omega_n)\right) \quad (6)$$

$$G(\mathbf{k}, i\omega_n) \approx \left(G_0(\mathbf{k}, i\omega_n)^{-1} - \overline{\Sigma}(i\omega_n)\right)^{-1} \quad (7)$$

which becomes very accurate in any of the following limits: large $\mid U \mid /t$; high temperatures; large spatial dimensions. Implicit in this formulation is the assumption that the self energy is largely $\mathbf{k}$ independent. We will discuss more fully the regimes of validity of this approximation in a future publication [6].

Representative results from our work are shown in two figures below. In the first, we show the energy dependence of the imaginary part of the self energy. The non self–consistent curve shows the seemingly unphysical result that the lifetime of the quasiparticles is actually *shortest* at the fermi energy. This result may be understood, at least in part, in terms the non self–consistent, non conserving theory of Ref. [7] — the fermion density is suppressed as one approaches the Thouless criterion, and all quasiparticles form two–particle bound states (this behaviour survives when one treats the non self–consistent theory in a conserving approximation [8]). Our simplified self–consistent, conserving theory shows that this physics is completely lost when interactions between the pairs are included — now one finds, similar to a usual fermi liquid, that the lifetimes of quasiparticles are *longest* at the fermi energy. Quite simply, this result reflects the absence of long–lived two–particle bound states in a self–consistent theory.

Perhaps the most interesting result which follows from this work involves the temperature dependence of the lifetime of quasiparticles. As shown in Fig. 2, we find that the inverse of this lifetime has a linear variation with temperature, similar to the experimentally observed inelastic scattering rate of the anomalous normal state of the high T_c superconductors [9]. The relationship of this result to different phenomenologies for the anomalous normal state will be presented elsewhere [6].

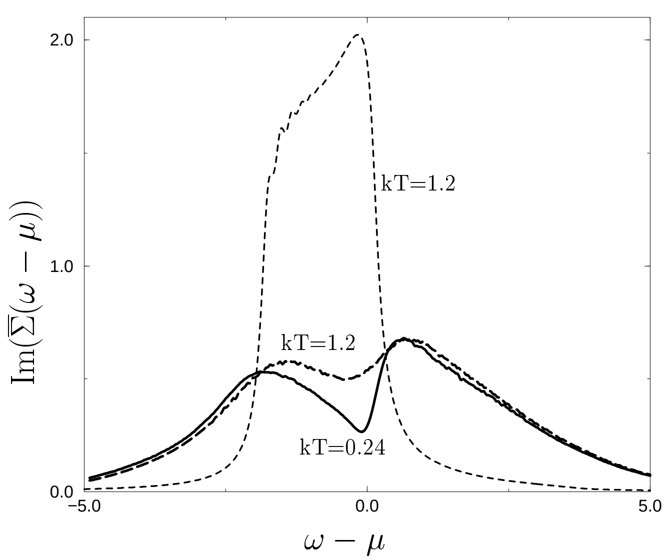

FIG. 1. The imaginary part of the self energy $\overline{\Sigma}(\omega-\mu)$ is plotted for two different temperatures, $kT = 1.2$ (dashed line) and $kT = 0.24$ (solid line) as a function of the frequency ω. For comparison we have plotted both the imaginary part of the non self–consistent self energy for $kT = 1.2$ (thin dashed line). All energies are in units of the transfer t with the model parameters given by $\mid U \mid /t = 8$, with electron filling $n = 0.2$.

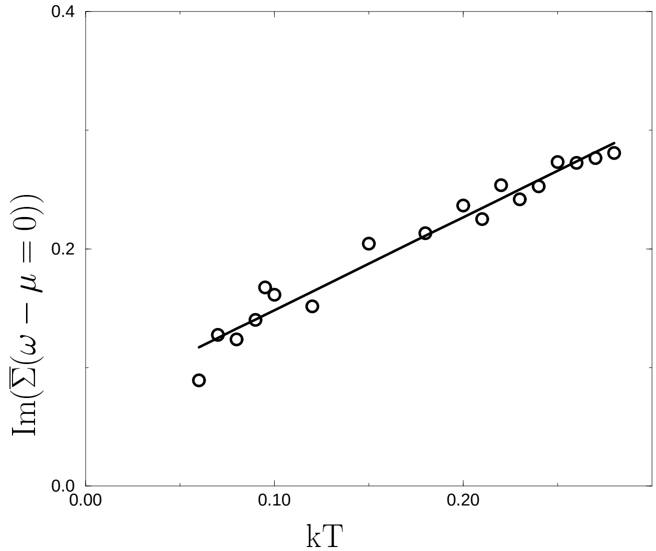

FIG. 2. The imaginary part of the self energy at the chemical potential, related to the inverse quasiparticle lifetime, is plotted as a function of temperature. A linear variation seems to interpolate through the numerical data quite well (solid line).

In conclusion, we have demonstrated the possibility of solving the attractive Hubbard model in 2–D in the ladder approximation self consistently by performing a k–average. Using this technique we include pair–pair interactions which hinder the formation of infinite lifetime bound states, and this leads to a quasiparticle lifetime that depends inversely on temperature, similar to many experiments.

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